Signals and Systems

Modules: Wideband True RMS Meter, Audio Oscillator, Adder, Utilities, Multiplier, Phase Shifter

0 Pre-Laboratory Reading

0.1 Root-Mean Square

The root-mean-square (rms) of a periodic signal $x(t)$ with period T is defined by

$$x_{\text{rms}} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} x^2(t)\, dt} \tag{1}$$

The relationship between x_{rms} and the peak value of $x(t)$ depends on the mathematical form of $x(t)$. For example:

$$\begin{aligned} x(t) &= C & \Longrightarrow & \quad x_{\text{rms}} = |C| \\ x(t) &= A \sin(2\pi f t) & \Longrightarrow & \quad x_{\text{rms}} = A/\sqrt{2}, \ A \geq 0 \end{aligned}$$

0.2 Decibels

The default scale for the Spectrum Mode is logarithmic with units dBu. This unit is defined as

$$\text{signal level} = 20 \log_{10} \left(\frac{V_{\text{rms}}}{0.775} \right) \text{ dBu} \tag{2}$$

where V_{rms} is the rms voltage and 0.775 V rms is a common reference voltage in audio electronics.

0.3 Sampling Spectrum Analyzer and Aliasing

The PicoScope in Spectrum Mode is a sampling spectrum analyzer. The sampling rate equals twice the frequency range.

$$\text{sampling rate} = 2 \times (\text{frequency range}) \tag{3}$$

These two (related) parameters define how aliasing happens.

Sampling creates copies of the spectrum of the analog signal that are displaced, within the frequency domain, from the original spectrum by integer multiples of the sampling rate. Denote the frequency range of a sampling spectrum analyzer by f_R. If the analog signal that is sampled has a frequency that lies between 0 and f_R, then this frequency will display exactly where you

would expect. If, however, the analog frequency is greater than f_R, it will alias to a lower frequency.

Suppose that the analog frequency equals $f_R + \Delta f$, where $0 < \Delta f < f_R$. The real sinusoid with this frequency has a Fourier transform with two Dirac delta functions: one at $f_R + \Delta f$ and one at $-(f_R + \Delta f)$. The second of these Dirac delta functions aliases to $2f_R - (f_R + \Delta f) = f_R - \Delta f$. (Remember that for a sampling spectrum analyzer the sampling rate is $2f_R$.) Of course, the only one of these frequencies that displays on the spectrum analyzer is the one that lies between 0 and f_R, and this is $f_R - \Delta f$. The picture below illustrates this. The analog frequency is $f_R + \Delta f$, but the frequency $f_R - \Delta f$ shows up on the display. One way of thinking about this aliasing is to say that the frequency axis folds at f_R, so that the analog frequency $f_R + \Delta f$ appears at $f_R - \Delta f$.
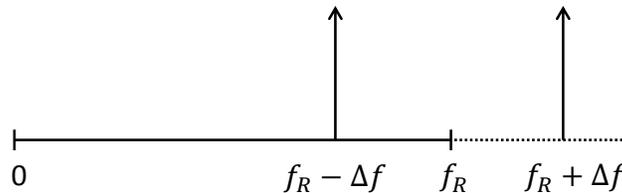
0.4 Response of Systems to a Sinusoid

The sinusoid is the single most important type of signal used to test the behavior of systems. The response of a system to a sinusoid depends on the type of system. The following table summarizes the most important facts.

System	Response to a sinusoid
Linear and time-invariant	Fundamental only (or nothing)
Nonlinear	Harmonics + DC
Even function	Even harmonics + DC
Odd function	Odd harmonics
Linear and time-varying	New frequencies, not harmonics

A linear, time-invariant (LTI) system responds to a sinusoid at its input by producing at its output either a sinusoid of the same frequency or nothing. (Filters are LTI systems that are typically designed to block sinusoids of certain frequencies, so the response of a filter to a sinusoid can be nothing.) There are never new frequencies on the output of an LTI system that were not also present on the input. Examples of LTI systems are: filters, phase shifters, and amplifiers operating in the linear regime.

A nonlinear system responds to a sinusoid at its input by producing at its output harmonics of the input sinusoid (including, possibly, the fundamental harmonic) and possibly a new DC component. If the characteristic input-output function of a nonlinear system is even, then only even harmonics and a DC component will be present on the output. If this function is odd, then

only odd harmonics will be present on the output. Examples of nonlinear systems are: limiters, rectifiers, square-law devices, and amplifiers operating in a nonlinear regime.

A linear, time-varying system responds to a sinusoid at its input by producing at its output sinusoids having new frequencies (not present on the input), but these new frequencies are not harmonics of the input sinusoid. An example of a linear, time-varying system is given later.

0.5 Square-Wave and Harmonics

A periodic signal $x(t)$ with period T has a Fourier series representation.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T} \tag{4}$$

where the a_k are the Fourier series coefficients. For a square-wave with an average value of zero, these coefficients have the magnitudes

$$|a_k| = \begin{cases} \dfrac{2A}{k\pi}, & k \text{ odd} \\[2mm] 0, & k \text{ even} \end{cases} \tag{5}$$

where A is the amplitude of the square-wave. This square-wave may be regarded as a sum of harmonics. The k-th harmonic is the sum of two terms in the Fourier series expansion: $a_k e^{j2\pi kt/T}$ and $a_{-k} e^{-j2\pi kt/T}$ (with k a positive integer). Using an Euler identity, it can be shown that the k-th harmonic is a sinusoid whose peak value is $2|a_k|$ and whose rms value is therefore $\sqrt{2}\,|a_k|$.

0.6 Multiplication of Sinusoids

When two sinusoids are multiplied together, the result is, in general, two sinusoids. From a trigonometric identity,

$$\cos(2\pi f_1 t)\cos(2\pi f_2 t) = \frac{1}{2}\cos[2\pi(f_1 - f_2)t] + \frac{1}{2}\cos[2\pi(f_1 + f_2)t] \tag{6}$$

In the special case where the two sinusoids being multiplied are the same,

$$\cos^2(2\pi f_1 t) = \frac{1}{2} + \frac{1}{2}\cos(4\pi f_1 t) \tag{7}$$

In this special case, the result is a DC component and a double-frequency sinusoid.

0.7 Example Linear, Time-Varying System

When a sinusoid is placed at the input of a linear, time-varying system, there will generally be new frequencies at the output. These new frequencies will not be harmonics of the input frequency.

The combination of the multiplier and oscillator shown below is an example of a linear but time-varying system. The equation governing this system is $y(t) = \cos(2\pi f_{LO}t)x(t)$. Note that at any moment in time the output is proportional to the input, making this a linear system. However, the coefficient, $\cos(2\pi f_{LO}t)$, is a function of time. This is a time-varying system. The frequency f_{LO} of the oscillator has the subscript LO as an indication that this is a local oscillator.
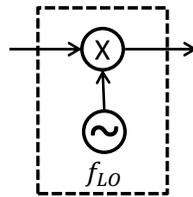
When the input is $\cos(2\pi f_I t)$, the output is

$$\cos(2\pi f_I t)\cos(2\pi f_{LO}t) = \frac{1}{2}\cos[2\pi(f_I - f_{LO})t] + \frac{1}{2}\cos[2\pi(f_I + f_{LO})t] \qquad (8)$$

In words, the input frequency f_I does not appear on the output; but two new frequencies, $f_I - f_{LO}$ and $f_I + f_{LO}$, appear on the output.

1 Scope Mode

In the Scope Mode, it is important to achieve a stable display with continuous capture of the signals. This is accomplished with triggering.

1.1 Sinusoid

A 100-kHz analog sinusoid is available on the Master Signals panel. The signal level is approximately 1.3 V rms. In this experiment, you will want to adjust the amplitude of this sinusoid, so you should connect it to a Buffer Amplifier and the output of this amplifier to both Channel A of the PicoScope (Scope Mode) and the Wideband True RMS Voltmeter.

 Channel A: 100-kHz sinusoid

You will stabilize the oscilloscope display by means of suitable triggering. The trigger mode should be set to *Auto*, and the trigger source to *A*.

Adjust the variable gain of the Buffer Amplifier so that the sinusoid at the amplifier output is first approximately 0.707 V rms and then approximately 1.414 V rms (as observed on the RMS Meter). In the first case you are using the Buffer Amplifier to attenuate the sinusoid, and in the second case you are amplifying the sinusoid. In both cases, use the oscilloscope to measure the peak voltage of the sinusoid. You can accurately measure the peak voltage using a signal ruler. There is a small rectangle at the top of the vertical scale; with the mouse, drag this rectangle down, creating a dashed line (a signal ruler). Align this signal ruler with the peak of the sinusoid. The ruler legend will indicate the peak voltage. Record the peak voltages, along with the ratio of the peak to rms voltage.

[x] Scope Mode sinusoid

Using the oscilloscope, measure the period of the sinusoid. The easy and accurate way to do this is to use time rulers. There is a small rectangle at the left end of the time axis; with the mouse, drag this rectangle to the right, creating a dashed vertical line (a time ruler). Place this time ruler at the location of one positive-going zero crossing. Place a second time ruler at the next positive-going zero crossing, so that the two time rulers together enclose one period. The ruler legend will indicate the period. The reciprocal of the period is the frequency. Separately, measure the frequency of the sinusoid using the Frequency Counter.

1.2 Two Sinusoids (Not Coherently Related)

The Master Signals panel also has an analog 2-kHz sinusoid. The frequency of this sinusoid is not exactly 2 kHz; it is actually 100 kHz divided by 48. Measure this frequency with the Frequency Counter.

Use the Audio Oscillator to generate a sinusoid having approximately the same frequency as the (100/48)-kHz sinusoid. You can use the Frequency Counter to measure the frequency of the Audio Oscillator output while adjusting the tuning knob on that oscillator.

Display the (100/48)-kHz sinusoid on Channel A and the Audio Oscillator output on Channel B. (Make sure that the range control is not set to *Off* for either channel.) Try using first Channel A as the trigger source, then Channel B. You should find that you can stabilize the display of one sinusoid (the one serving as the trigger source) but not both simultaneously. The reason for this is that the two sinusoids are not (coherently) derived from a common oscillator with the frequency of one sinusoid an integer multiple of the other.

Channel A: (100/48)-kHz sinusoid
Channel B: 2-kHz sinusoid from Audio Oscillator

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It is always possible to freeze the display by clicking on *Stop*. But that stops signal capture. You could change a signal (or even remove it from the oscilloscope), and the oscilloscope would not reflect this change when its display is stopped.

We generally want continuous capture of signals and also stable displays. With two or more signals, this is only possible if the signals are coherently related.

1.3 Two Sinusoids (Coherently Related)

Place the 100-kHz analog sinusoid from the Master Signals panel on Channel A. Place the (100/48)-kHz analog sinusoid (from the Master Signals panel) on Channel B. Notice that the frequency in Channel A is an integer (48) multiple of the frequency on Channel B. We say that these two sinusoids are coherently related. Try using first Channel A as the trigger source, then Channel B. You should find that you can stabilize the display of both sinusoids simultaneously only if the (100/48)-kHz sinusoid is the trigger source. This confirms the following general oscilloscope rule: *Two sinusoids can be simultaneously stabilized only if each is coherently related to the trigger source and the frequency of each is a whole-number multiple of the frequency of the trigger source.* In the present case, if we use the (100/48)-kHz sinusoid as the trigger source, the whole number multiples are 100 (Channel A) and 1 (Channel B).

   Channel A: 100-kHz sinusoid
Channel B: (100/48)-kHz sinusoid

1.4 TTL Signals

Place the 100-kHz analog sinusoid on Channel A. Place the 8.3-kHz TTL signal (which actually has a fundamental frequency of 100 kHz divided by 12) on Channel B. You should be able to get a stable display of both the 100-kHz analog sinusoid and the 8.3-kHz TTL signal simultaneously by using the 8.3-kHz TTL signal as the trigger source. When using a TTL signal as the trigger source, the trigger level should be great than 0 V and less that 5 V. It is suggested that you use a trigger level of 2 V.

   Channel A: 100-kHz sinusoid
Channel B: (100/12)-kHz TTL signal

2 Spectrum Mode

When in Spectrum Mode, the PicoScope is a sampling spectrum analyzer. Aliasing becomes an issue.

Use the VCO module to generate a sinewave. Before inserting this module into the TIMS instrument, make sure that the slide switch on the PCB is in the VCO position. Set the front-

panel toggle switch to HI. Using the tuning knob (the lower knob), adjust the frequency to approximately 70 kHz, as measured on the Frequency Counter.

Place the 70-kHz sinusoid from the VCO at the input of a Buffer Amplifier. Connect the amplifier output to Channel A of the PicoScope (Spectrum Mode), to the Wideband True RMS Voltmeter, and to the Frequency Counter. Adjust the amplifier gain for an amplifier output of 0.775 V rms. This corresponds to 0 dBu. Set the spectrum analyzer frequency range to 98 kHz (so that the sampling rate is 196 kHz). You should see a strong line near 70 kHz with a line height of approximately 0 dBu.

📊 Channel A: VCO output

Now change the frequency range to 49 kHz; the sinusoid will now appear at an alias frequency, but its line height will remain the same. You can measure the alias frequency accurately in Spectrum Mode. Click on *Add Measurement* (the plus sign at the bottom of the PicoScope display), and then select the measurement *Frequency at Peak*. Repeat this procedure for a frequency range of 24 kHz. You should notice that the Frequency Counter in the TIMS instrument always measures an analog frequency of approximately 70 kHz. The aliasing happens within the PicoScope card (the sampling spectrum analyzer).

📊 Spectrum Mode

3 Characterization of Systems

In the following series of experiments, you will characterize a number of systems. The XY View is a useful oscilloscope technique for studying the input-output relationships of systems.

3.1 Buffer Amplifier

Place the 2-kHz sinusoid from the Master Signals panel on the input of a Buffer Amplifier. Observe the amplifier input on Channel A and the amplifier output on Channel B. You will notice that the Buffer Amplifier is an inverting amplifier. (That is, the output is 180° out of phase with the input.) Adjust the amplification so that the voltage gain magnitude is 2. (The peak voltage out of the amplifier is twice that at the input.) Make sure the range control is set the same for Channels A and B (for example, both ± 5 V) so that there is a common vertical scale for both signals.

📈 Channel A: input of Buffer Amplifier
Channel B: output of Buffer Amplifier

Switch to the XY View.

`Views > X-Axis > A`

You are now viewing the amplifier output as a function of the input. The slope is negative because it is an inverting amplifier.

◯   Channel A: input of Buffer Amplifier
   Channel B: output of Buffer Amplifier

You can return to viewing the Channel A and B signals as a function of time.

`Views > X-Axis > Time`

3.2 Weighted Adder

The module called Adder is actually a weighted adder. That is, if the inputs are $x_1(t)$ and $x_2(t)$, the output is $y(t) = G_1 x_1(t) + G_2 x_2(t)$, where G_1 and G_2 are weighting factors that are adjusted with knobs on the Adder module. Both G_1 and G_2 are negative. Dialing the G_1 knob clockwise increases its absolute value, but G_1 remains negative, and similarly for G_2.

Place the 2-kHz sinusoid from the Master Signals panel on one input of the Adder. Leave the second input open. Observe the Adder input on Channel A and the Adder output on Channel B. You will notice that the output is $180°$ out of phase with the input. Adjust the weighting factor so that the voltage gain magnitude is 2.

   Channel A: input of Adder
   Channel B: output of Adder

Switch to the XY View. Note the slope of the displayed line.

◯   Channel A: input of Adder
   Channel B: output of Adder

3.3 Cascaded Buffer Amplifiers

Sometimes you will want amplification without the inversion. You can achieve this by cascading two Buffer Amplifiers together. Place the 2-kHz (Master Signals) sinusoid on the input of the first Buffer Amplifier and the output of this amplifier on the input to the second. Observe the unamplified 2-kHz sinusoid on Channel A and the output of the cascaded amplifiers on Channel B. Adjust the amplification, using the knobs on both amplifiers, until the cascaded voltage gain is approximately 4. You will note that the overall gain is non-inverting.

   Channel A: input of first Buffer Amplifier
   Channel B: output of cascaded Buffer Amplifiers

Switch to the XY View. The slope should now be positive.

⬭ Channel A: input of first Buffer Amplifier
   Channel B: output of cascaded Buffer Amplifiers

Place a copy of the cascaded amplifier output on the RMS Meter input. Adjust the cascaded gain until the output is approximately 13 V rms. At this level, the cascaded amplifier is no longer operating in the linear regime; it must now be regarded as a nonlinear system. This nonlinear characteristic curve has odd symmetry, so you should expect that this system will generate odd, but not even, harmonics. Switch back to viewing the cascaded amplifier output as a function of time and observe the distortion caused by nonlinear operation.

〰 Channel A: input of first Buffer Amplifier
   Channel B: output of cascaded Buffer Amplifiers (nonlinear operation)

Select the Spectrum Mode and display just the output of the cascaded amplifier. You should see the fundamental (approximately 2 kHz) and some other odd harmonics. Adjust the frequency range so that you get a good view of the harmonics. If the vertical scale is logarithmic, you will see even harmonics, but they will be small. If an odd harmonic is larger than a neighboring even harmonic by 30 dB, then the power in the odd harmonic is larger than that in the even harmonic by three orders of magnitude. Select a linear vertical scale (click on the Spectrum options icon), and the domination of the odd harmonics will be clearer.

📊 Channel B: output of cascaded Buffer Amplifiers

Make a table showing the rms voltage V_k for the k-th harmonic as a function of k. You should use a linear scale, which displays rms voltage.

📇 cascaded Buffer Amplifiers

You should experiment with decreasing the gain of the cascaded amplifiers. As the gain and output level decreases, the higher-order harmonics should decrease. If you decrease the gain enough, the higher-order harmonics will disappear, leaving only the fundamental. This indicates that the system is now in linear operation.

3.4 Hard Limiter

You will use the comparator on the Utilities module as a hard limiter. Place the 2-kHz sinusoid on the analog input of the comparator and also on Channel A. Connect the reference input of the comparator to ground (located on the Variable DC panel). Place the TTL output of the comparator on Channel B. It is important that you set Channel B coupling to AC. (A TTL

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signal has a DC value, but this DC component is undesired in the present experiment. AC coupling will block the DC component.) View the Channel A and B signals simultaneously. You will see that the 2-kHz sinusoid has been turned into a square-wave of the same frequency.

Channel A: input of hard limiter
Channel B: output of hard limiter

This is the action of the hard limiter. A positive input produces an output $+A$, and a negative input produces an output $-A$. (In the present case, A is approximately 2.5 V.) Hence, an input sinusoid produces a square-wave of amplitude A at the limiter output. This is a useful function in certain communication applications where the only information about a sinusoid that is needed is the zero-crossings.

Switch to the XY View. You will see that the input-output characteristic function is nonlinear and odd. You should expect that the output therefore contains only odd harmonics (and including the fundamental).

Channel A: input of hard limiter
Channel B: output of hard limiter

Select the Spectrum Mode and display only the output of the limiter, making sure that the coupling is AC. Use a linear scale.

Channel B: output of hard limiter

Make a table with the expected rms voltage and the measured rms voltage for each of the several strongest harmonics. The expected voltage will come from the recognition that a square-wave has a Fourier series representation. The expected voltage for the k-th harmonic will be $\sqrt{2}\,|a_k|$, where a_k is a Fourier series coefficient.

hard limiter

3.5 Half-Wave Rectifier

An ideal half-wave rectifier would pass the positive half cycle of an input sinusoid and would null the negative half cycle. A practical half-wave rectifier, like the one in the Utilities module that you will use in this experiment, does not completely null the negative half cycle.

Place the 2-kHz sinusoid at the input of the rectifier and also on Channel A. Place the rectifier output on Channel B. Observe the effect of the rectifier on an input sinusoid.

 Channel A: input of rectifier
Channel B: output of rectifier

Then switch to the XY View. Even though this characteristic function has a linear portion, this is a nonlinear system because of the abrupt change in slope at the origin.

 Channel A: input of rectifier
Channel B: output of rectifier

Select the Spectrum Mode. Observing the output of the rectifier, you should see a DC component, a fundamental harmonic, and higher-order harmonics. Because the characteristic curve is neither odd nor even, there should be both even and odd harmonics present.

 Channel B: output of rectifier

3.6 Phase Shifter

A phase shifter is an LTI system that delays an input signal. The response of a phase shifter to a sinusoid is a sinusoid of the same frequency but with a phase shift. The Phase Shifter module has a slide switch on its PCB. Make sure this switch is set to "HI".

Place the 100-kHz analog sinusoid at the input of the Phase Shifter and on Channel A. Place the output of the Phase Shifter on Channel B. Add an XY View.

Views > Add View > XY

You should now have two displays: the Channel A and B signals as a function of time and an XY View of these two signals. Adjust the delay until the output lags the input by $90°$. If you have trouble reaching a lag of $90°$, toggle the switch on the front of the Phase Shifter.

 Channel A: input of Phase Shifter
 Channel B: output of Phase Shifter

You can recognize a $90°$ delay in both displays. In the time display, the output will have a positive-going zero-crossing at the same moment that the input is at its maximum. In the XY View, you will see an ellipse. (If the amplitudes of the input and the delayed sinusoids were equal and if the aspect ratio of the display were 1:1, the XY View would look like a perfect circle when the phase shift is $\pm 90°$. However, the aspect ratio here is not 1:1, so the displayed figure is an ellipse.) If the phase difference between the two sinusoids is truly $\pm 90°$, the one axis of the ellipse will be horizontal and the other vertical. You should note that the XY View does not tell you which sinusoid is leading and which is lagging.

3.7 Linear, Time-Varying System

Place the (100/48)-kHz analog sinusoid at the input to the Multiplier. Place the 100-kHz analog sinusoid at the other input. Place the Multiplier output on Channel A. If the oscillator (inside the Master Signals panel) that produces the 100-kHz sinusoid and the Multiplier are considered the system, then this is a linear, time-varying system with the (100/48)-kHz sinusoid as its input.

Observe the Multiplier output as a function of time. In order to stabilize this display, you should use a (100/48)-kHz TTL signal as an external trigger source. If you are using the older version of the TIMS-301C instrument, you will have to generate a (100/48)-kHz TTL signal from the (100/12)-kHz TTL signal using a divide-by-4 on the Digital Utilities module.
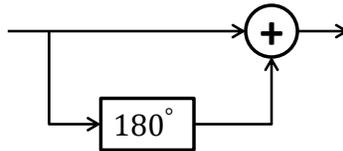
     Channel A: Multiplier output

Select the Spectrum Mode. Note the frequencies that are present. These will hopefully be the frequencies that you expected to see.

     Channel A: Multiplier output

4 Nulling a Sinusoid

In communications engineering, there is an occasional need to null a sinusoid. This can be accomplished as indicated here:

It is essential to realize that a good null can be achieved only if two conditions are met. First, the phase shift must be very close to 180°. Second, the amplitude of the original sinusoid must be equal to that of the inverted copy.

As a first step, connect the 100-kHz analog sinusoid to the input of the Phase Shifter. Connect the input and output of the Phase Shifter to the oscilloscope. Use the XY View to help you adjust the phase shift for 180°. You are looking for the ellipse to collapse to a straight line with a negative slope.

     Channel A: input of Phase Shifter
     Channel B: output of Phase Shifter

You will be using the weighted adder (called Adder). Even if the amplitudes of the original sinusoid and its inverted copy are identical, you will not get a null if the two weighting factors are not equal. You will use the following technique. With only the original sinusoid connected to the weighted adder, you will adjust its weighting factor so that the output is 1.00 V rms. Then you will disconnect the original sinusoid from the weighted adder and connect the inverted sinusoid to the other input of the weighted adder. You will adjust the weighting factor for the inverted sinusoid so that the output is again 1.00 V rms. Finally, you will reconnect the original sinusoid back into the weighted adder, so that the circuit is complete.

Place the original 100-kHz sinusoid on Channel A and the output of the Adder on Channel B. If you have done the nulling procedure correctly, the signal on Channel B should be small. It probably won't be exactly zero because the phase shift is probably not exactly $180°$ and there is probably not a perfect amplitude match between the original and inverted sinusoids. Change the range control for Channel B to ±100 mV. Try to make the Channel B signal as small as possible. You will do this in a two-step procedure. First, use the fine control on the Phase Shifter to try to make the output of the nulling circuit smaller yet. Second, adjust the weighting factor of either the original or the inverted sinusoid (but not both) slightly. Hopefully, with these two steps you can get a little better null.

Channel A: 100-kHz sinusoid
Channel B: output of Adder